Multiplicative-Accumulative matching of NLO calculations with parton showers

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NLO + shower matching methods

	MC@NLO	POWHEG	KrkNLO
applicability	any shower	any shower	only showers with shower real > NLO real everywhere
1st step of shower	shower prog.	NLO prog.	shower prog.
negative weights?	intrinsic	largely absent	absent(?)

Valuable to have more than one NLO+shower matching method

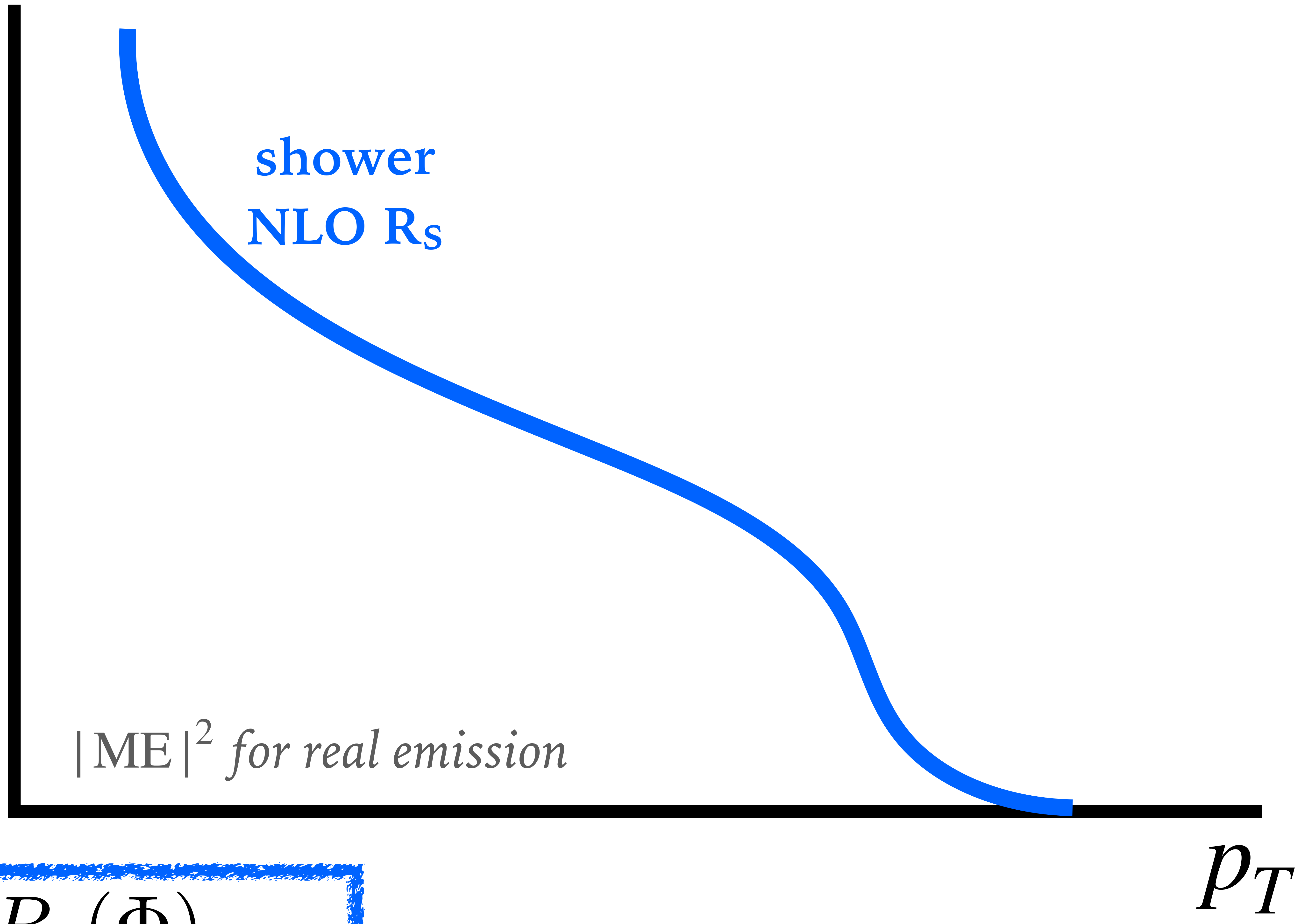
Maybe valuable to have NLO+shower matching where shower has full showering control (e.g. ongoing logarithmic-accuracy work from PanScales, Manchester-Vienna, Deductor)

But do we have to live with negative weights?

MC@NLO

Generate "Born" events (Φ_B) with NLO normalisation, $\bar{B}_s/B_0$

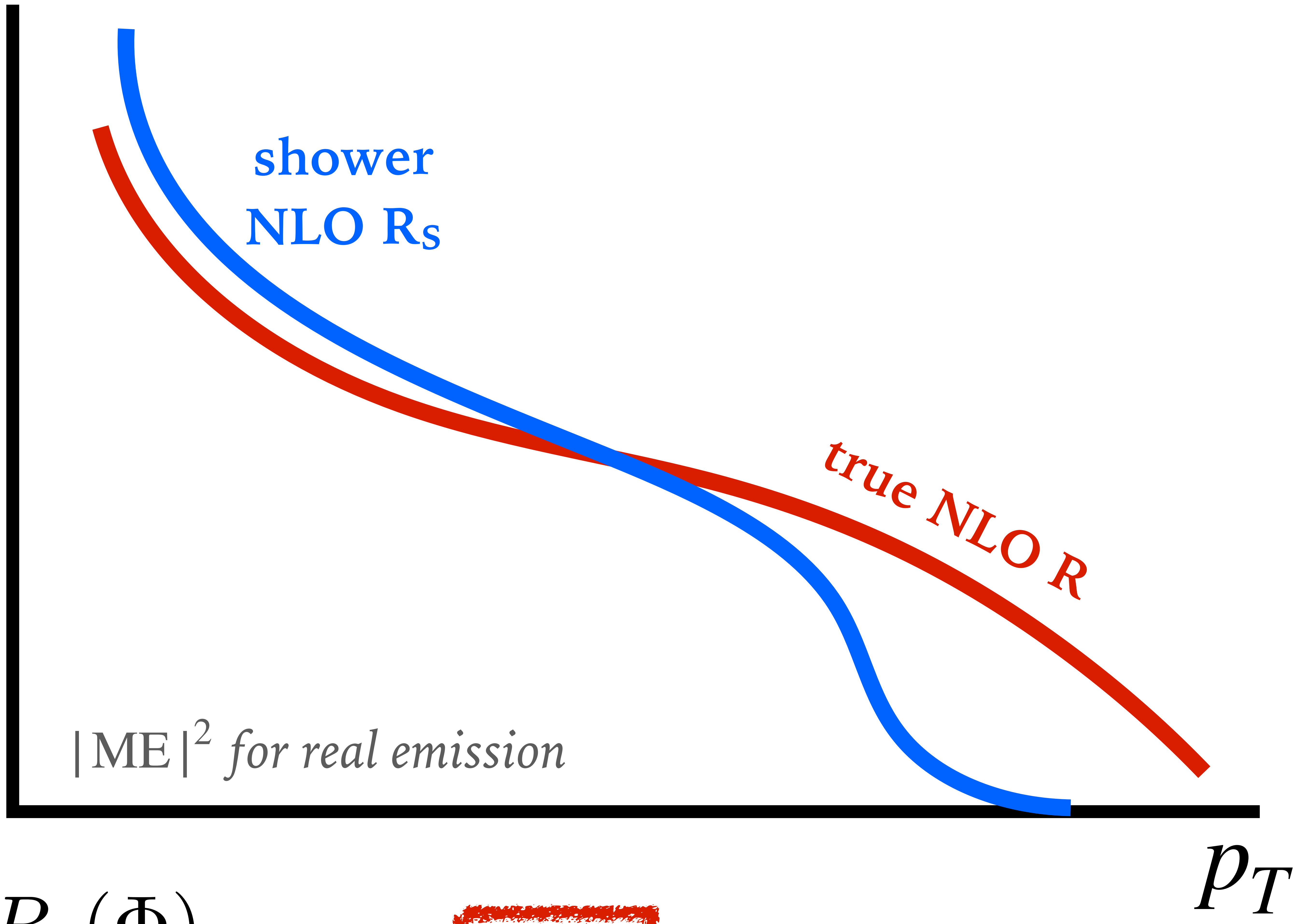
Let Pythia/Herwig/Sherpa shower them (combining Sudakov $S(t,\Phi)$ & real shower radiation R_s)



$$d\sigma = \bar{B}_s(\Phi_B)\, S(t_\Phi, \Phi_B) \times \frac{R_s(\Phi)}{B_0(\Phi_B)} d\Phi + \left[R(\Phi) - R_s(\Phi)\right] d\Phi$$

MC@NLO

True real radiation matrix-element, R, differs from shower R_s



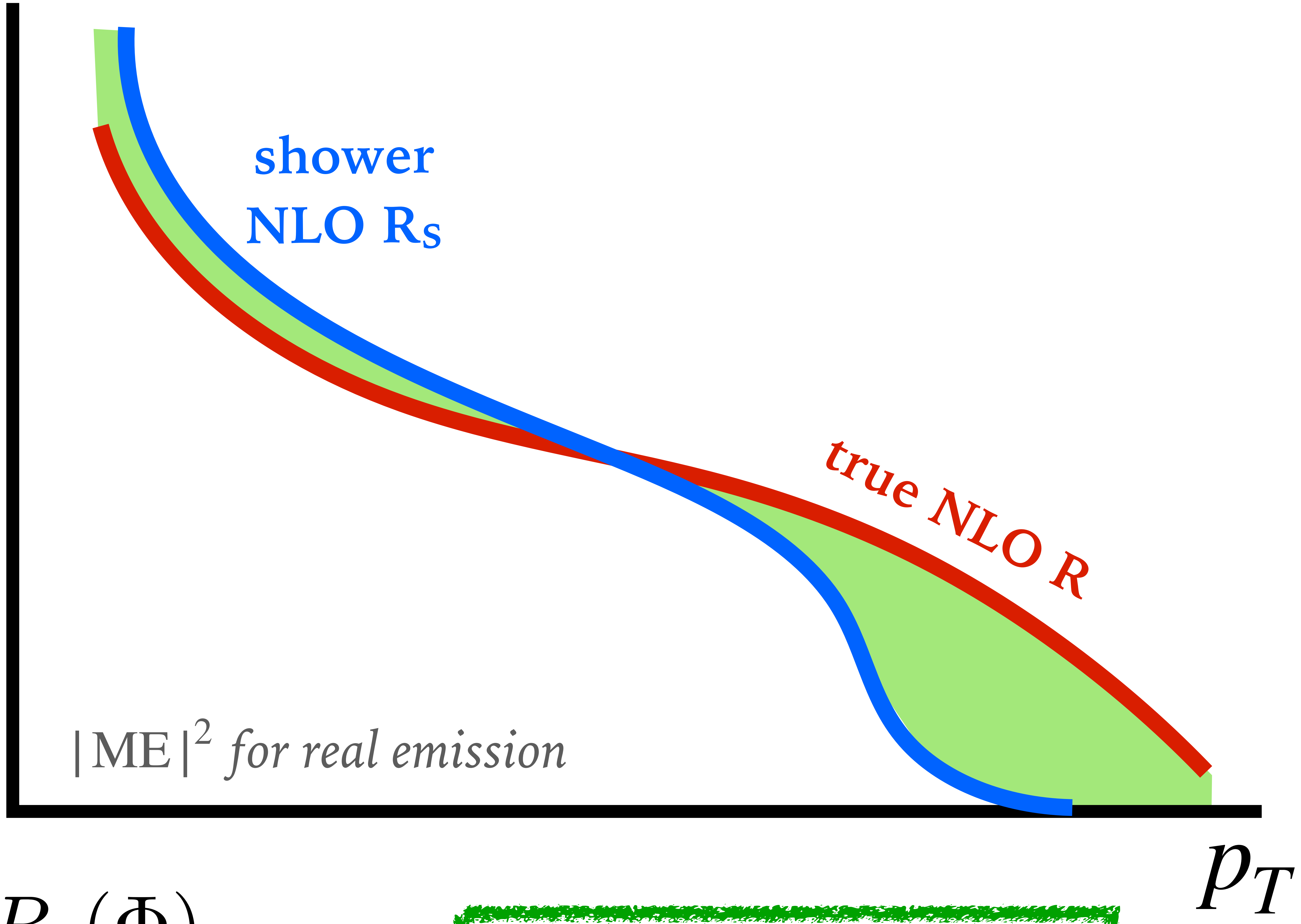
$$d\sigma = \bar{B}_\mathrm{S}(\Phi_\mathrm{B})\, S(t_\Phi, \Phi_\mathrm{B}) \times \frac{R_\mathrm{S}(\Phi)}{B_0(\Phi_\mathrm{B})} d\Phi + [R(\Phi) - R_\mathrm{S}(\Phi)]\, d\Phi$$

MC@NLO

True real radiation matrix-element, R, differs from shower R_s

correct for this difference by adding a sample of real events with weights $R(\Phi) - R_s(\Phi)$

(and shower them)



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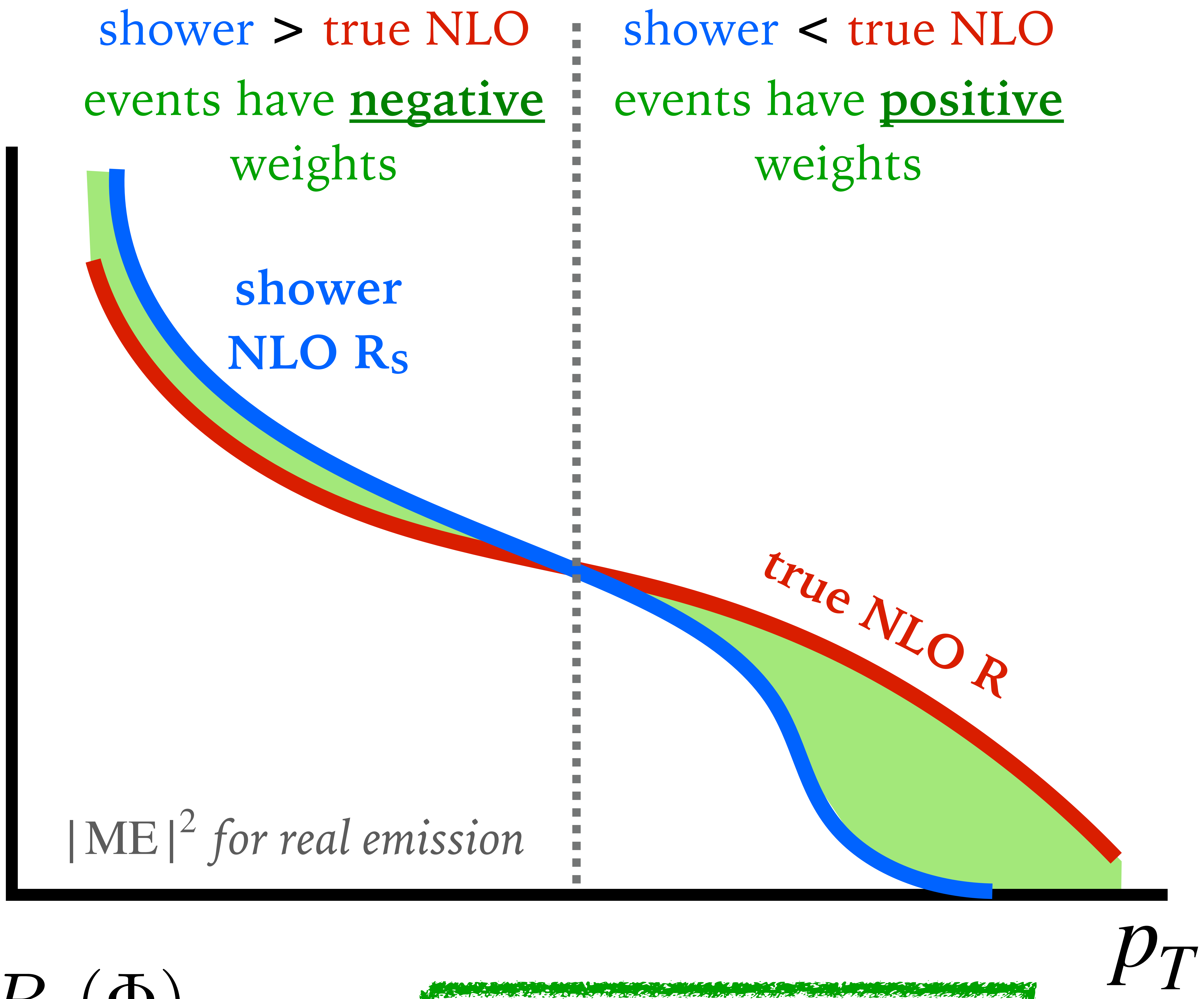

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MC@NLO

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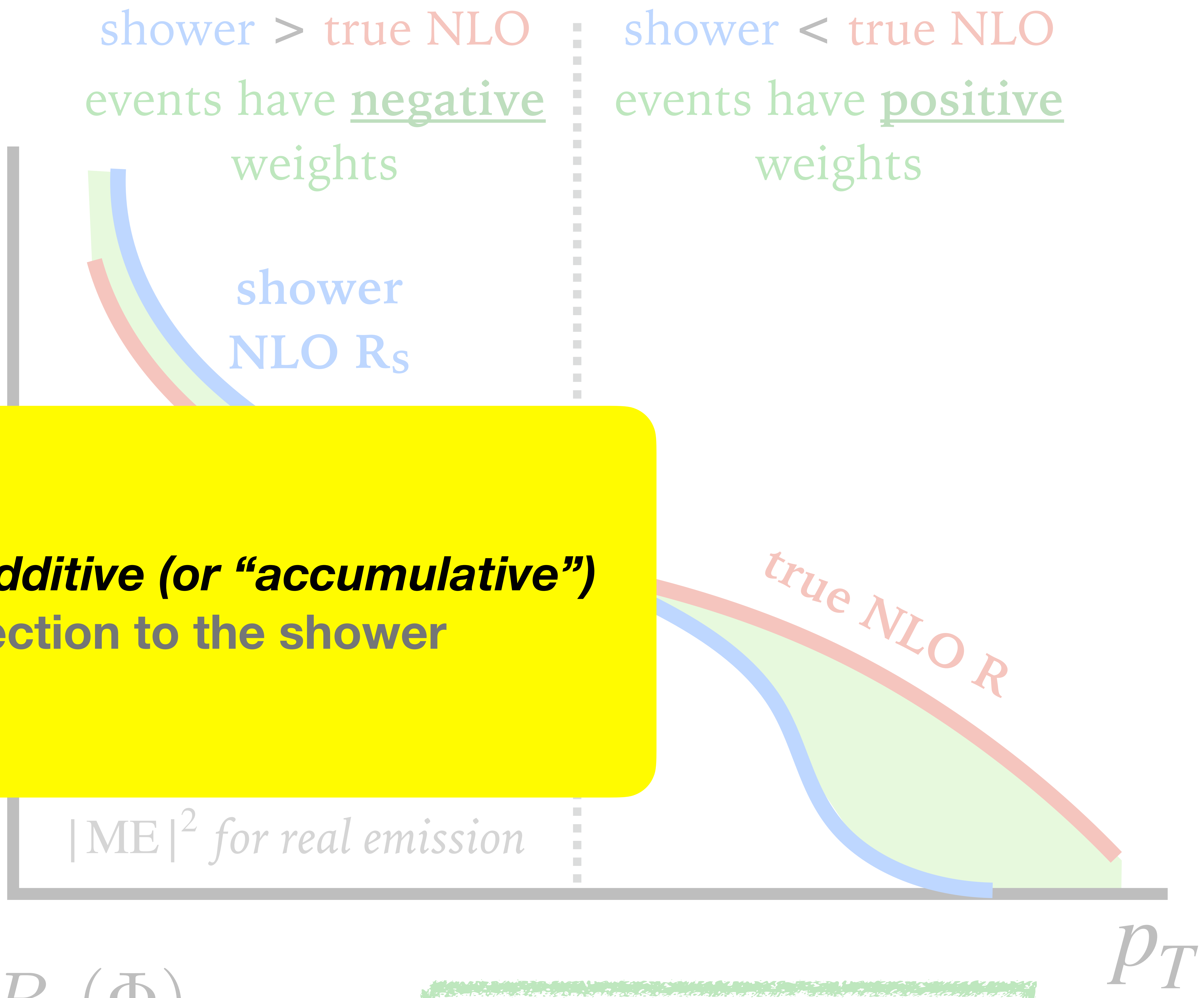


This is an *additive (or "accumulative")* correction to the shower

$$d\sigma = \bar{B}_s(\Phi_B)\, S(t_\Phi, \Phi_B) \times \frac{R_s(\Phi)}{B_0(\Phi_B)} d\Phi + [R(\Phi) - R_s(\Phi)]\, d\Phi$$

KrkNLO

"KrkNLO" papers, 2015 onwards from Jadach, Nail, Płaczek, Sapeta, Siódmok and Skrzypek, …, pointed out (among various other things) the following.

If the shower satisfies property that $R_s(\Phi) > R(\Phi)$ for all phase-space points, you can replace additive matching (and their negative weights) by "**multiplicative**" matching: you multiply the effective shower event weight, $R_s(\Phi)$, by

$$\frac{R(\Phi)}{R_S(\Phi)}$$

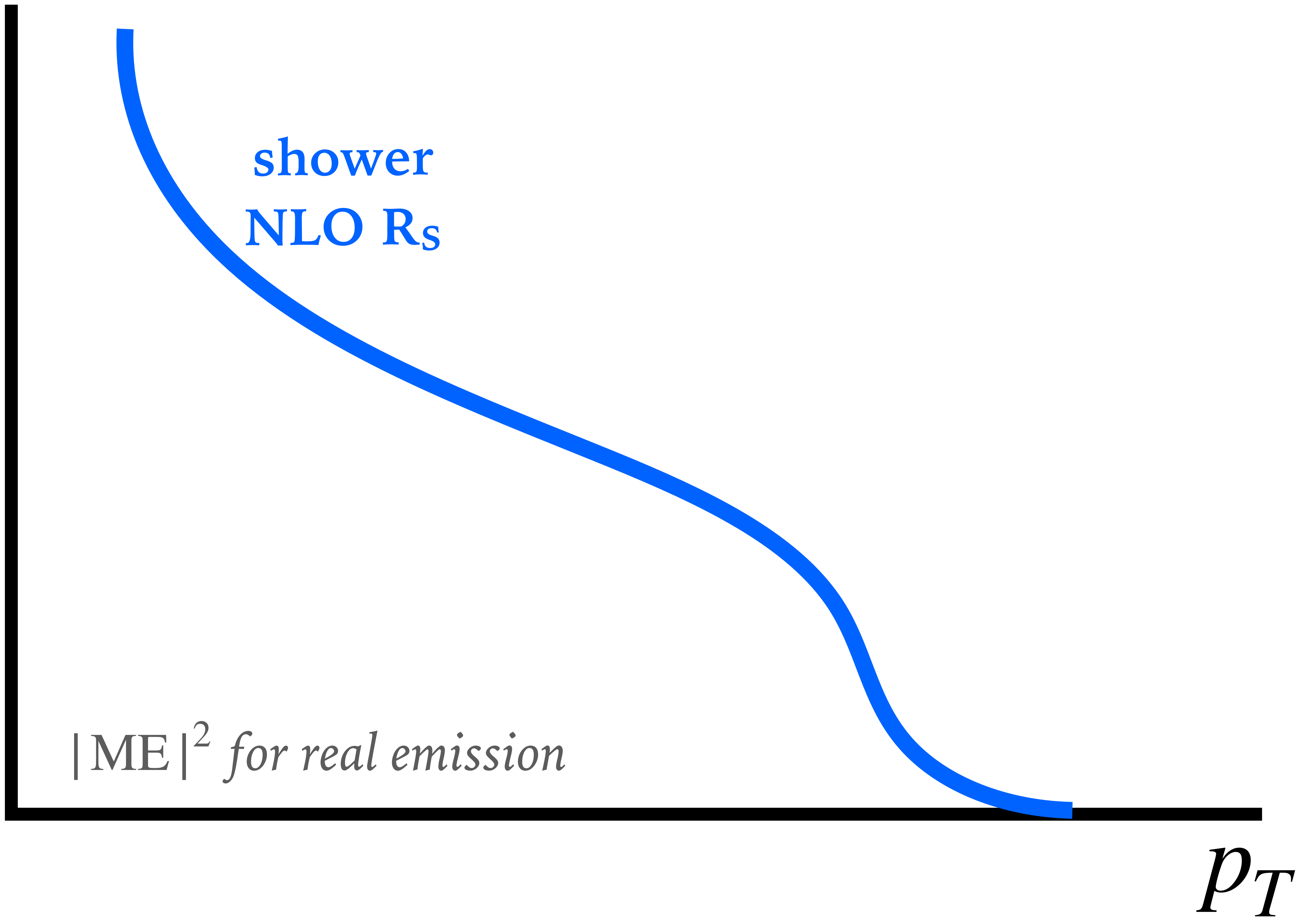
which you implement by accepting the showered event with probability $R(\Phi)/R_s(\Phi)$.

$$d\sigma = \bar{B}_\mathrm{S}(\Phi_\mathrm{B}) \left\{ S(t_\Phi, \Phi_\mathrm{B}) \times \frac{R_\mathrm{S}(\Phi)}{B_0(\Phi_\mathrm{B})} \right\} \times \left[\frac{R(\Phi)}{R_\mathrm{S}(\Phi)} \right] d\Phi$$

Core idea of 2111.03553

As with MC@NLO:
generate "Born" events (Φ_B)
with NLO normalisation, $\bar{B}_s/B_0$

Let Pythia/Herwig/Sherpa shower them
(combining Sudakov $S(t,\Phi)$ &
real shower radiation R_s)



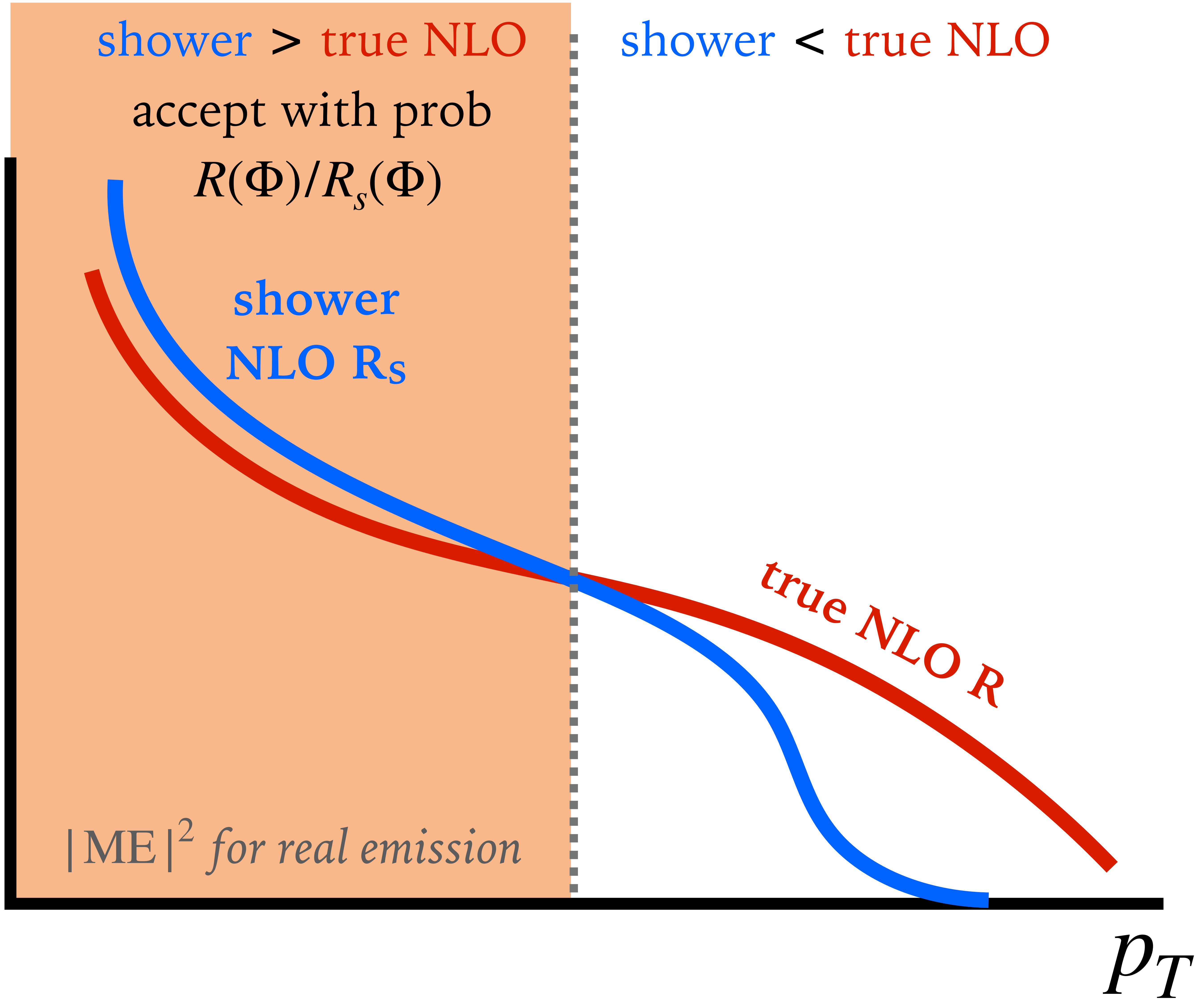
$$d\sigma = \bar{B}_\mathrm{s}(\Phi_\mathrm{B})\, S(t_\Phi,\Phi_\mathrm{B}) \times \frac{R_\mathrm{s}(\Phi)}{B_0(\Phi_\mathrm{B})} \times \left\{1 + \frac{R - R_\mathrm{s}}{R_\mathrm{s}}\theta(R_\mathrm{s} - R)\right\} d\Phi + \theta(R - R_\mathrm{s})\,[R - R_\mathrm{s}]\, d\Phi$$

Core idea of 2111.03553

After one or more steps of the showering of the Born events, determine Φ (1-emission phase-space point).

If $R(\Phi) < R_s(\Phi)$, accept the event with probability $R(\Phi)/R_s(\phi)$

(otherwise always accept event)



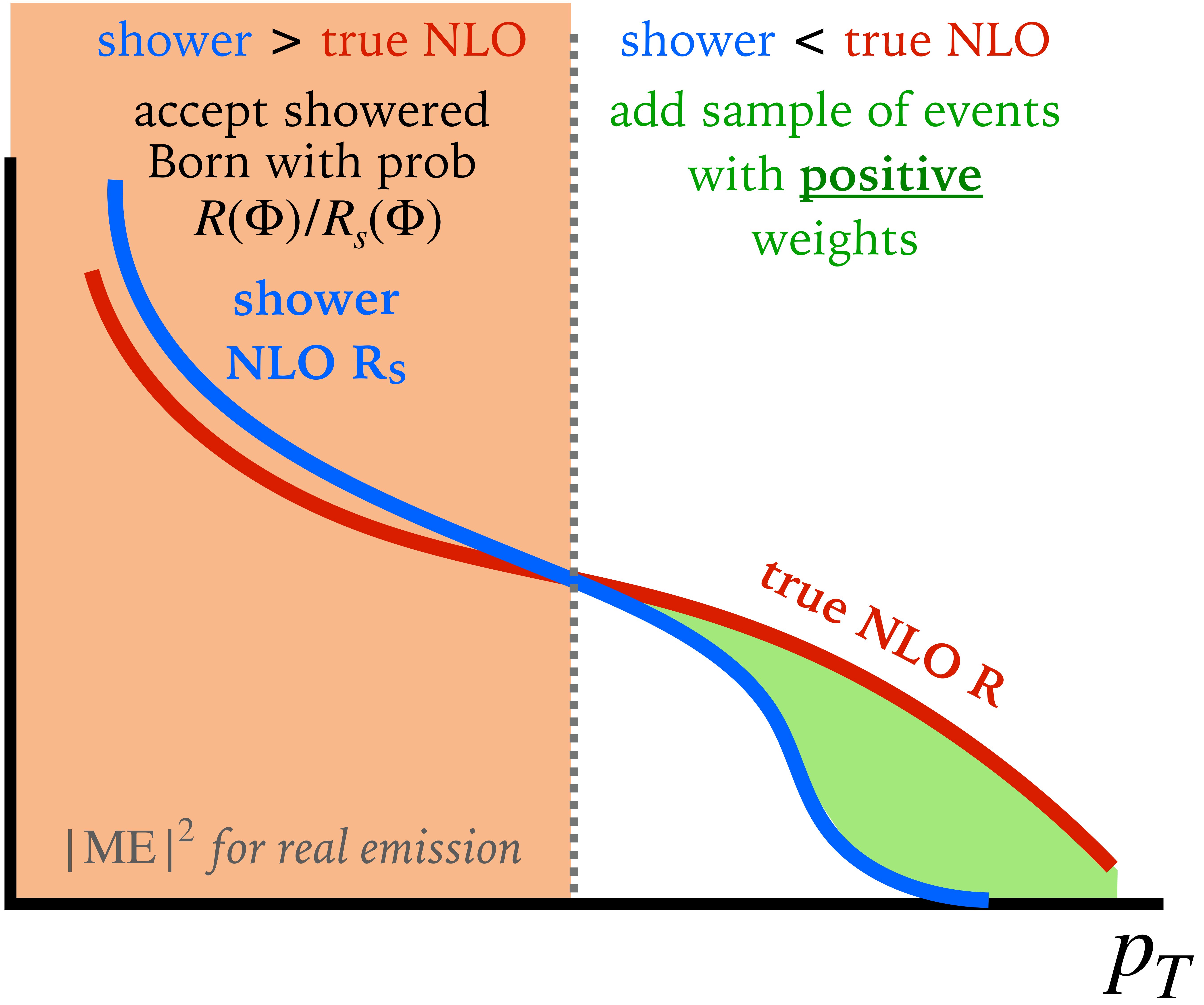
$$d\sigma = \bar{B}_s(\Phi_B)\, S(t_\Phi, \Phi_B) \times \frac{R_s(\Phi)}{B_0(\Phi_B)} \times \left\{ 1 + \frac{R - R_s}{R_s} \theta(R_s - R) \right\} d\Phi + \theta(R - R_s)\, [R - R_s]\, d\Phi$$

Core idea of 2111.03553

Add in sample of positive-weight "real" events where shower is an underestimate, i.e. additively correct regions where

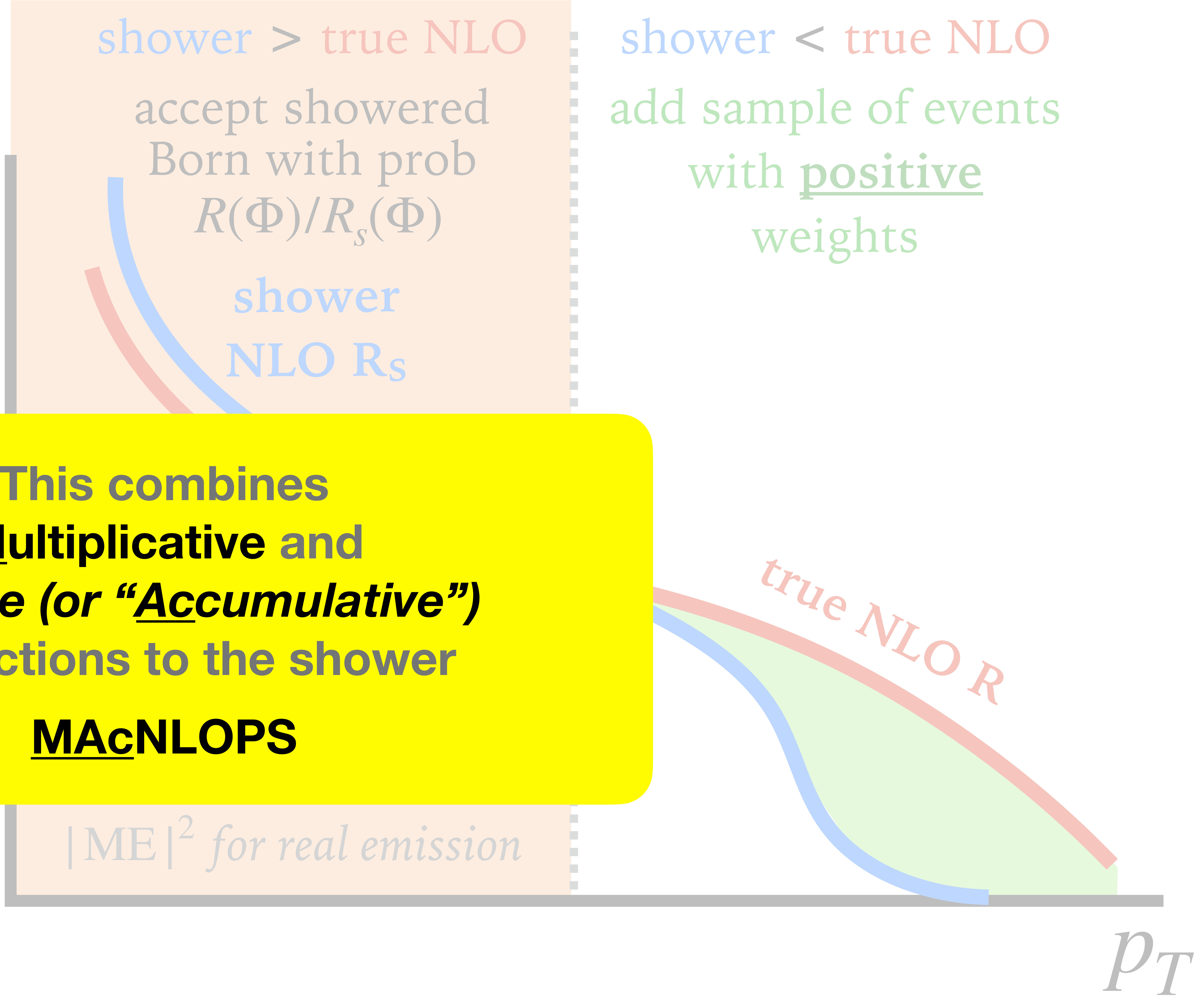
$R(\Phi) > R_s(\Phi)$

(and shower them)



$$d\sigma = \bar{B}_s(\Phi_B)\, S(t_\Phi, \Phi_B) \times \frac{R_s(\Phi)}{B_0(\Phi_B)} \times \left\{1 + \frac{R - R_s}{R_s}\theta(R_s - R)\right\} d\Phi + \theta(R - R_s)\,[R - R_s]\, d\Phi$$

Core idea of 2111.03553



Add in sample of positive
“real” events to correct
where $R(\Phi) > R_s($
(and shower them)

This combines
<u>M</u>ultiplicative and
additive (or “<u>Ac</u>cumulative”)
corrections to the shower

<u>MAc</u>NLOPS

$$d\sigma = \bar{B}_s(\Phi_B)\, S(t_\Phi, \Phi_B) \times \frac{R_s(\Phi)}{B_0(\Phi_B)} \times \left\{1 + \frac{R - R_s}{R_s}\theta(R_s - R)\right\} d\Phi + \theta(R - R_s)\,[R - R_s]\, d\Phi$$

operational workflow

MCatNLO

Generate sample of LHE **Born** events
(weights specific to chosen shower)

Generate sample of LHE **Real** (+/-) events
$R(\Phi) - R_s(\Phi)$
(specific to chosen shower)

shower both samples with chosen shower

operational workflow

MCatNLO

Generate sample of LHE **Born** events
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$R(\Phi) - R_s(\Phi)$
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shower both samples with chosen shower

POWHEG

Generate sample of LHE **Real** (+) events, according to POWHEG first "shower" step

continue showering them with chosen shower, using event-specific starting scale

operational workflow

MCatNLO

Generate sample of LHE **Born** events
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POWHEG

Generate sample of LHE **Real** (+) events, according to POWHEG first "shower" step

continue showering them with chosen shower, using event-specific starting scale

MAcNLOPS (+p_T-ordered shower)

Generate sample of LHE **Born** events
(weights specific to chosen shower)

let chosen shower do first emission

if $R(\Phi) < R_s(\Phi)$ reject event with prob
$1 - R(\Phi)/R_s(\Phi)$

let shower continue

Generate sample of LHE **Real** (+) events
$R(\Phi) - R_s(\Phi) > 0$
(specific to chosen shower)

shower with chosen shower

operational workflow

MCatNLO

- Generate sample of LHE **Born** events (weights specific to chosen shower)
- Generate sample of LHE **Real** (+/-) events $R(\Phi) - R_s(\Phi)$ (specific to chosen shower)
- shower both samples with chosen shower

POWHEG

- Generate sample of LHE **Real** (+) events, according to POWHEG first “shower” step
- continue showering them with chosen shower, using event-specific starting scale

MAcNLOPS (+p_T-ordered shower)

- Generate sample of LHE **Born** events (weights specific to chosen shower)
- let chosen shower do first emission
- if $R(\Phi) < R_s(\Phi)$ reject event with prob $1 - R(\Phi)/R_s(\Phi)$
- let shower continue

- Generate sample of LHE **Real** (+) events $R(\Phi) - R_s(\Phi) > 0$ (specific to chosen shower)
- shower with chosen shower

Shower codes often include hooks to allow this kind of operation + must be linked with real matrix element

MAcNLOPS variant #1: over-generate Born events by some factor c > 1

Generate sample of LHE **Born** events, with number of events enhanced by factor c
(weights specific to chosen shower)

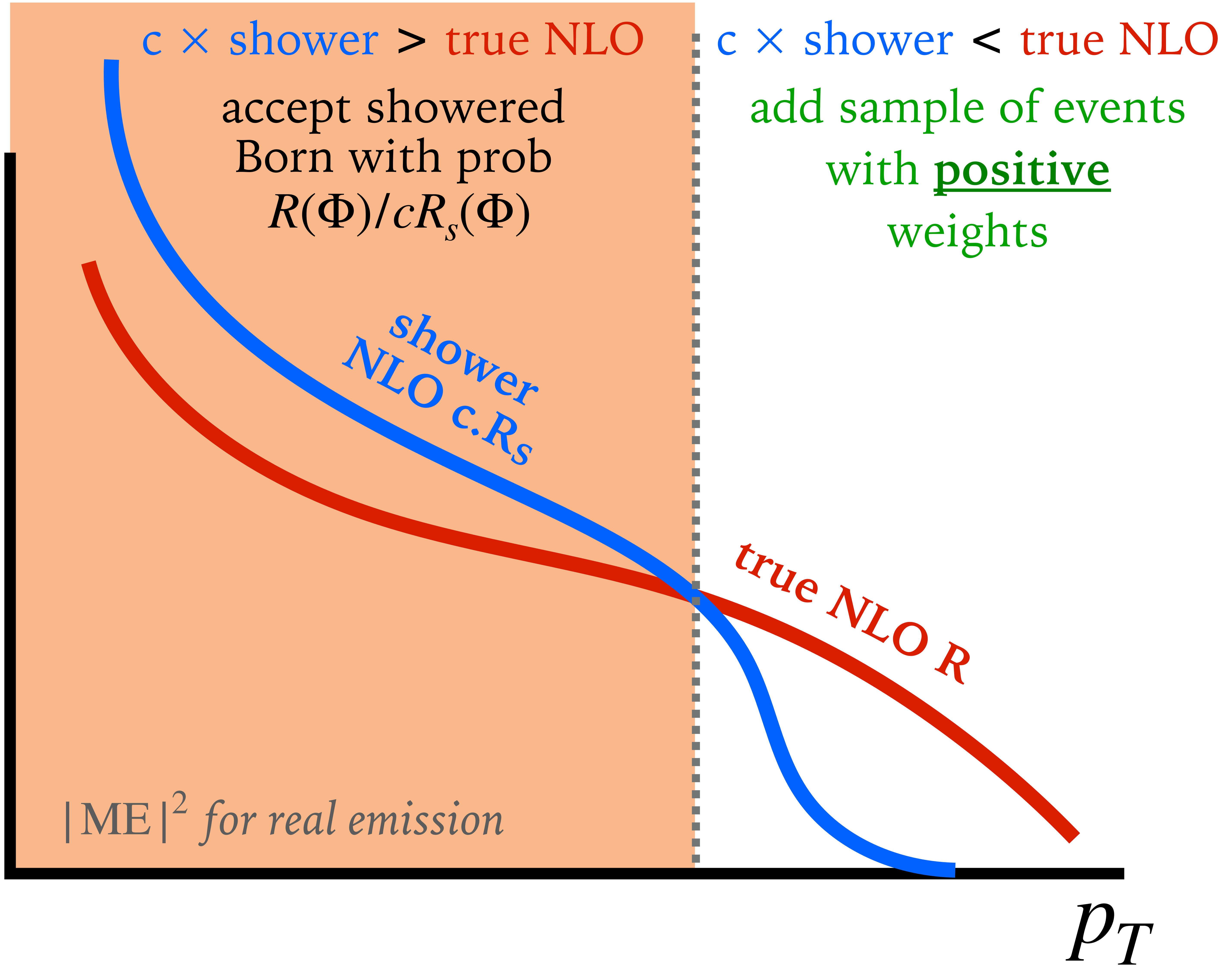
let chosen shower do first emission

if $R(\Phi) < cR_s(\Phi)$ reject event with prob $1 - R(\Phi)/(cR_s(\Phi))$

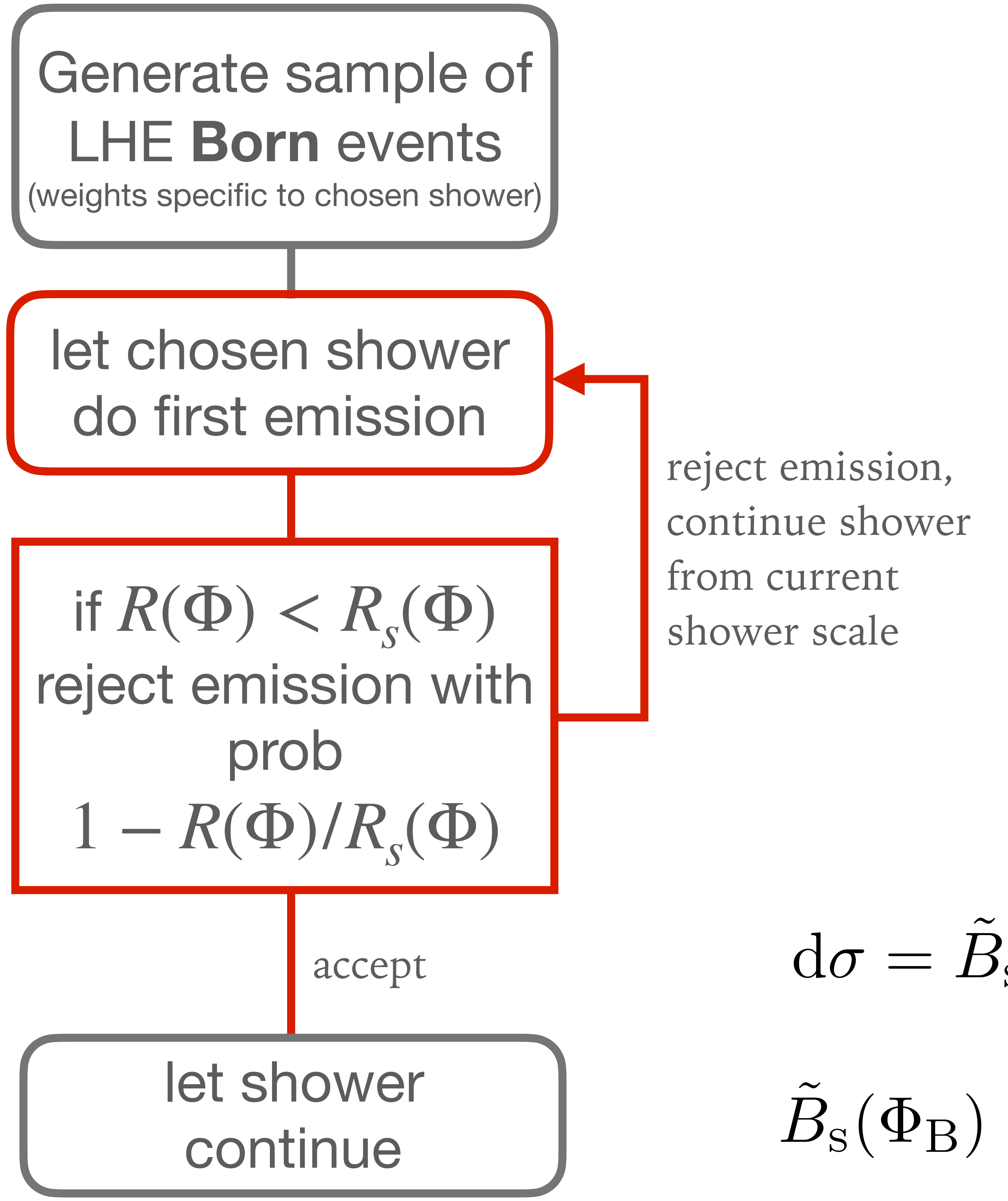
let shower continue

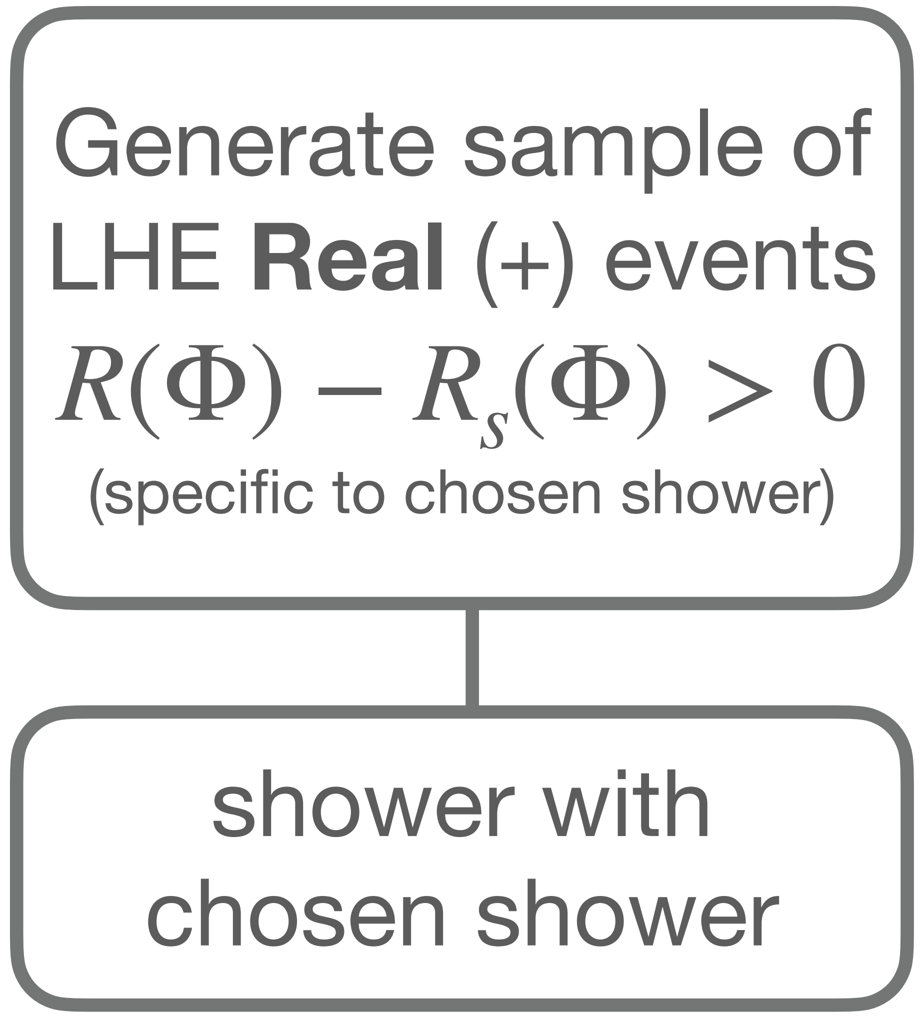
Generate sample of LHE **Real** (+) events, $R(\Phi) - cR_s(\Phi) > 0$
(specific to chosen shower)

shower with chosen shower



MAcNLOPS variant #2: reject first emission, not whole event





like MCatNLO and POWHEG, this variant maintains the standard NLO cross section

$$d\sigma = \tilde{B}_{\rm s}(\Phi_{\rm B})\,\tilde{S}(t_\Phi, \Phi_{\rm B}) \times \frac{\min(R(\Phi), R_{\rm s}(\Phi))}{B_0(\Phi_{\rm B})} d\Phi + \theta(R - R_{\rm s})\,[R - R_{\rm s}]\,d\Phi$$

$$\tilde{B}_{\rm s}(\Phi_{\rm B}) = B_0(\Phi_{\rm B}) + V(\Phi_{\rm B}) + \int \min[R(\Phi), R_{\rm s}(\Phi)] d\Phi_{\rm rad}$$

Conclusions

- There are various ways to match NLO and shower beyond canonical MC@NLO / POWHEG pair
- New Multiplicative-Accumulate family (MAcNLOPS) leaves responsibility for the shower with the shower program, like MC@NLO, while avoiding its limitation(?) of irreducible negative weights
- Should be straightforward to implement,
  - uses same ingredients already available in MC@NLO
  - shower program needs to link with real matrix elements in order to calculate rejection probability for events (or emissions)
  - further care needed with angular-ordered showers (identification of effective Φ_R phase space point when shower not ordered in hardness)

backup

$$\bar{B}_{\rm s}(\Phi_{\rm B}) = B_0(\Phi_{\rm B}) + V(\Phi_{\rm B}) + \int R_{\rm s}(\Phi) {\rm d}\Phi_{\rm rad},$$

$$S(t, \Phi_{\rm B}) = \exp\left[-\int_{t_\Phi > t} \frac{R_{\rm s}(\Phi)}{B_0(\Phi_{\rm B})} {\rm d}\Phi_{\rm rad}\right]$$

$$d\sigma = \bar{B}_{\rm s}(\Phi_{\rm B})\, S(t_\Phi, \Phi_{\rm B}) \times \frac{R_{\rm s}(\Phi)}{B_0(\Phi_{\rm B})} {\rm d}\Phi + [R(\Phi) - R_{\rm s}(\Phi)]\, {\rm d}\Phi$$

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$$d\sigma = \bar{B}_{\rm s}(\Phi_{\rm B})\, S(t_\Phi, \Phi_{\rm B}) \times \frac{R_{\rm s}(\Phi)}{B_0(\Phi_{\rm B})} \times \left\{1 + \frac{R - R_{\rm s}}{R_{\rm s}} \theta(R_{\rm s} - R)\right\} {\rm d}\Phi + \theta(R - R_{\rm s})\, [R - R_{\rm s}]\, {\rm d}\Phi$$

$$d\sigma = \bar{B}_{\rm s}(\Phi_{\rm B})\, S(t_\Phi, \Phi_{\rm B}) \times \frac{c\, R_{\rm s}(\Phi)}{B_0(\Phi_{\rm B})} \times \left\{1 + \frac{R - c\, R_{\rm s}}{c\, R_{\rm s}} \theta(c\, R_{\rm s} - R)\right\} {\rm d}\Phi +$$
$$+\, \theta(R - c\, R_{\rm s})\, [R - c\, R_{\rm s}]\, {\rm d}\Phi$$

$$\tilde{B}_{\rm s}(\Phi_{\rm B}) = B_0(\Phi_{\rm B}) + V(\Phi_{\rm B}) + \int \min[R(\Phi), R_{\rm s}(\Phi)] {\rm d}\Phi_{\rm rad}$$

$$d\sigma = \tilde{B}_{\rm s}(\Phi_{\rm B})\, \tilde{S}(t_\Phi, \Phi_{\rm B}) \times \frac{\min(R(\Phi), R_{\rm s}(\Phi))}{B_0(\Phi_{\rm B})} {\rm d}\Phi + \theta(R - R_{\rm s})\, [R - R_{\rm s}]\, {\rm d}\Phi$$

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